Active Filter Circuits

1. Active filter can produce bandpass and bandreject filters without using inductors (costly, heavy, large, noisy)
   ↳ may introduce electromagnetic field effects that compromise the desired frequency response characteristics.

2. Filtering + Amplification
   Active filters provide a control over amplification not available in passive filter circuits.

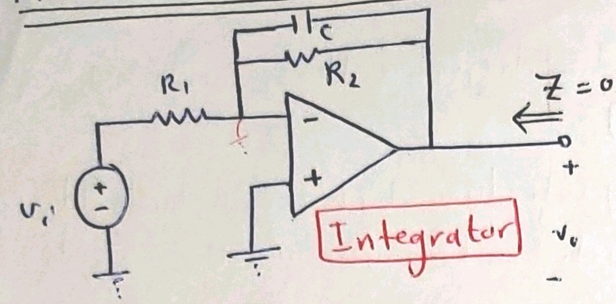
3. The cutoff frequency and the passband magnitude of passive filters were altered with the addition of a resistive load at the output of the filter (R_L).

⇨ We use active circuits to implement filter designs when gain, load variation, and physical size are important parameters in the design specifications.

Active Filters

- First Order
  - Low-Pass
  - High-Pass
- Higher Order

First-Order Low-Pass Active Filter :-



$\frac{1}{Cs} \Rightarrow$ open

$w = 0$ amp with gain $-\frac{R_2}{R_1}$

$w = \infty$ $V_o = 0$

$$H(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

where $Z_2(s) = R_2 \parallel \frac{1}{Cs}$

$Z_1(s) = R_1$

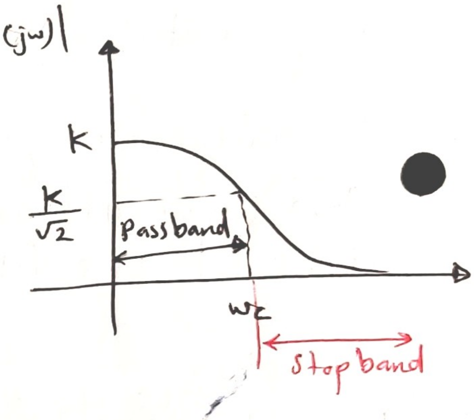
$$= -\frac{R_2}{R_1} \frac{1/R_2C}{s + 1/R_2C}$$

$$H(s) = -K \frac{w_c}{s + w_c}$$ (low-pass filter)

where $\boxed{K = \frac{R_2}{R_1} \quad w_c = \frac{1}{R_2 C}}$ (Pass band gain) may be > 1 (cut off frequency)

In design

R_2 ... ما بنغير

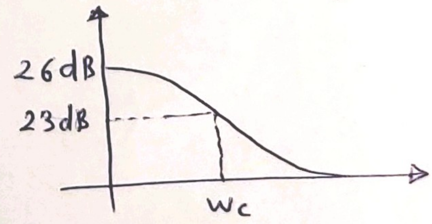


<u>note</u>

$$20\log_{10}\left|\frac{1}{\sqrt{2}}\right| = -3\,dB$$

cutoff freq $\equiv$ the frequency at which the maximum magnitude of the transfer function in dB has been reduced by 3dB.

<u>example</u>



$26 - 3 = 23\,dB$

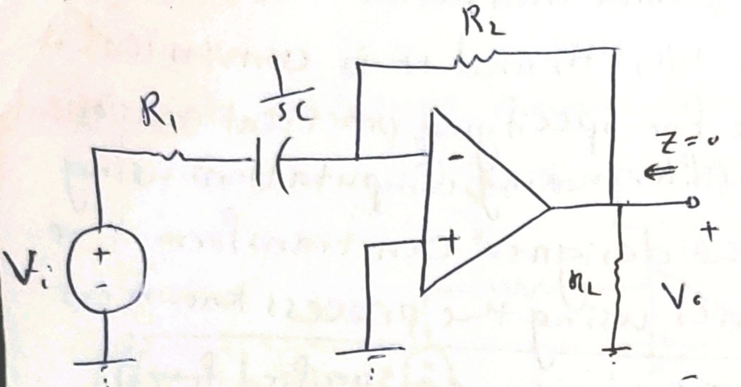
the mag used to find the cutoff frequency.

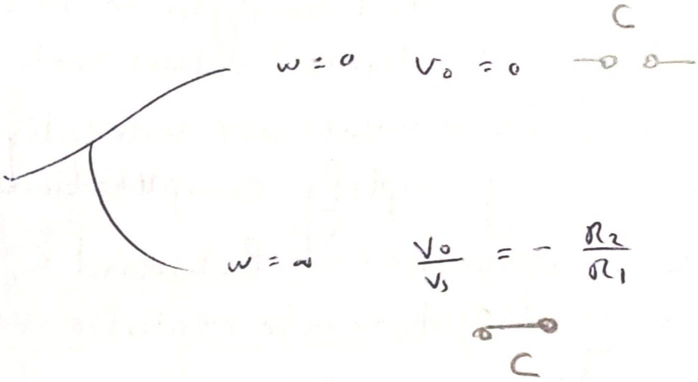
Or

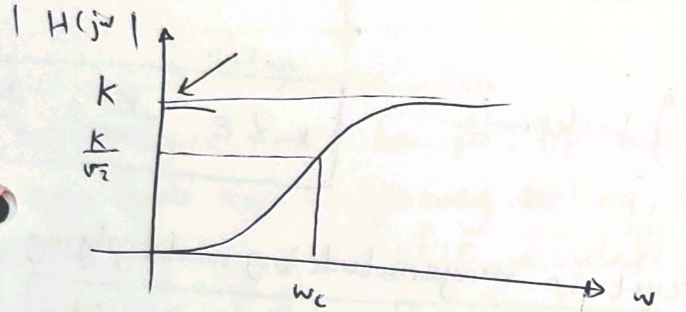
$20\log x = 26 \Rightarrow x = 19.95$

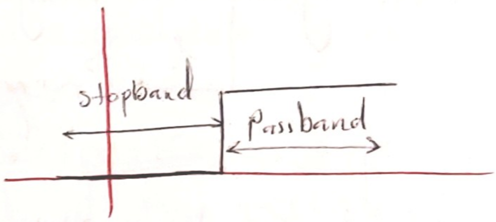
$x/\sqrt{2} = 14.1 \Rightarrow 20\log 14.1 = 23$ ok

2) First-Order High-Pass Active Filter :









$$H(s) = \frac{-Z_2(s)}{Z_1(s)}$$

where

$$Z_2 = R_2$$

$$Z_1 = R_1 + \frac{1}{sC}$$

$$H(s) = -\frac{R_2}{R_1} \frac{s}{s + \frac{1}{R_1 C}}$$

$$H(s) = -K \frac{s}{s + w_c}$$

$$\boxed{\begin{aligned} K &= \frac{R_2}{R_1} \\ w_c &= \frac{1}{R_1 C} \end{aligned}}$$

(Passband gain) may be > 1

مهم

In design

ورتأثرو بـ RL

Scaling:

In the design and analysis of both passive and active filter circuits, working with element values such as 1Ω, 1 H, and 1F is convenient. Although these values are unrealistic for specifying practical components, they greatly simplify ~~computations~~ computations. After making computations using convenient values of R, L, and C, the designer can transform the convenient values into realistic values using the process known as $\boxed{\text{scaling}}$.

((T.F the same + specified freq))

Scaling → magnitude. / frequency. — Independent

note:- $k_m Z = \frac{k_m}{j\omega C} = \frac{1}{j\omega \frac{C}{k_m}}$

- magnitude scaling: We scale a circuit in maganitude by multiplying the impedance at a given freq by the scale factor km.

$\grave{R} = k_m R \quad , \quad \grave{L} = k_m L$

$\grave{C} = C / k_m$

$$\boxed{\omega_c' = \frac{1}{R_2' C'} = \frac{1}{R_2 k_m \cdot \frac{C}{k_m}} = \frac{1}{R_2 C}}$$

$$\boxed{k = \frac{R_2'}{R_1'} = \frac{R_2 k_m}{R_1 k_m} = \frac{R_2}{R_1}}$$

km: positive real number that can be either less than or greater than 1.
((mag scale factor))

- Frequency Scaling:-

In freq scaling, we change the ckT parameters so that at the new freq, the impedance of each element is the same as it was at the original frequency.

$\omega_1 \longrightarrow \omega_2$

$\omega_2 = k_f \, \omega_1$

* $\grave{R} = R$

* $j\omega_2 \grave{L} = j\omega_1 L$

$j k_f \omega_1 \grave{L} = j\omega_1 L$

$$\boxed{\grave{L} = \frac{L}{k_f}}$$

$\frac{1}{j\omega_2 \grave{C}} = \frac{1}{j\omega_1 C}$

$\frac{1}{j k_f \omega_1 \grave{C}} = \frac{1}{j\omega_1 C}$

$$\boxed{\grave{C} = \frac{C}{k_f}}$$

① $\grave{R} = k_m R$

② $\grave{L} = \frac{k_m}{k_f} L$

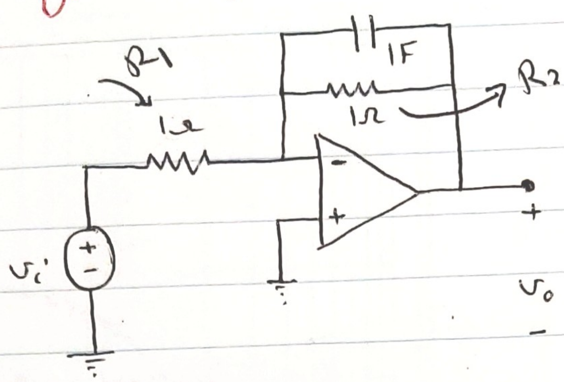
③ $\grave{C} = \frac{1}{k_m k_f} C$

④ $k = k$

⑤ $\omega_c' = k_f \omega_c \quad , \quad \omega_o' = k_f \omega_o$

The frequency scale factor kf is also a positive real number that can be less than or greater than unity.

Example :- prototype low-pass filter :-



$H(s) = -K \frac{w_c}{s+w_c}$

$\boxed{H(s) = \frac{-1}{s+1}}$

$k = \frac{R_2}{R_1}$

$w_c = 1$ rad/sec $\qquad w_c = \frac{1}{R_2 C}$

Use the prototype low-pass opamp filter from example 1, along with magnitude and frequency scaling, to compute the R values for a low-pass filter with a gain of 5, a cutoff frequency of 1000 Hz, and a feedback capacitor of 0.01 μF. Construct a Bode plot of the resulting transfer function's magnitude.

k_m, k_f ?!

$\boxed{2\pi f = \text{rad/sec} \atop w}$

[1] $w_c = 1000\text{ Hz} = 1000(2\pi)$

$w_c' = k_f w_c \Rightarrow k_f = \frac{w_c'}{w_c} = \frac{2\pi(1000)}{1} = 6283.1853$

[2] $C' = \frac{1}{k_m k_f} C \qquad C' = 1 * 10^{-8}$ F

$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{1}{(6283.1853)(1*10^{-8})} = 15915.5$

[3] $R_1 = R_2 = 1\,\Omega$

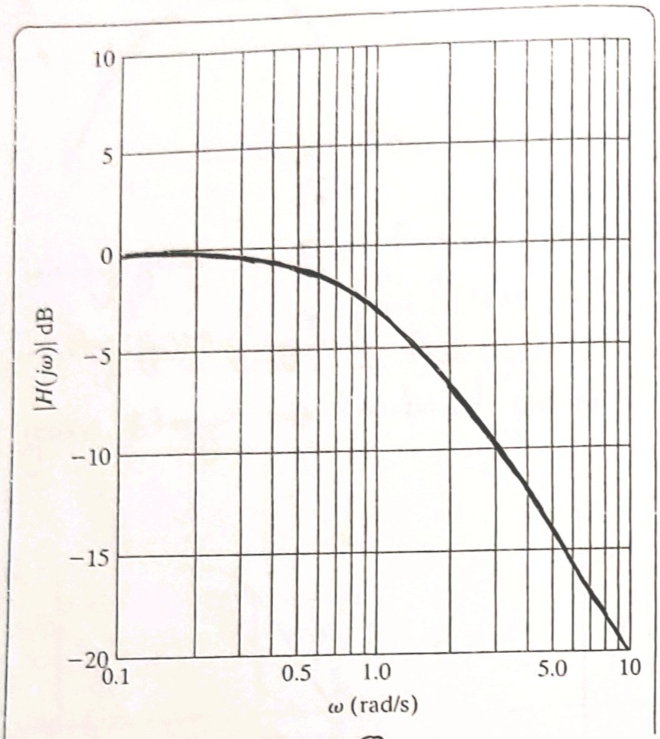
$R_1' = R_2' = k_m R = (15915.5)(1) = 15915.5\,\Omega$

[4] $K = \frac{R_2}{R_1}$, $w_c = \frac{1}{R_2 C}$ If we adjust R_2, we will change the cutoff frequency.

gain 5

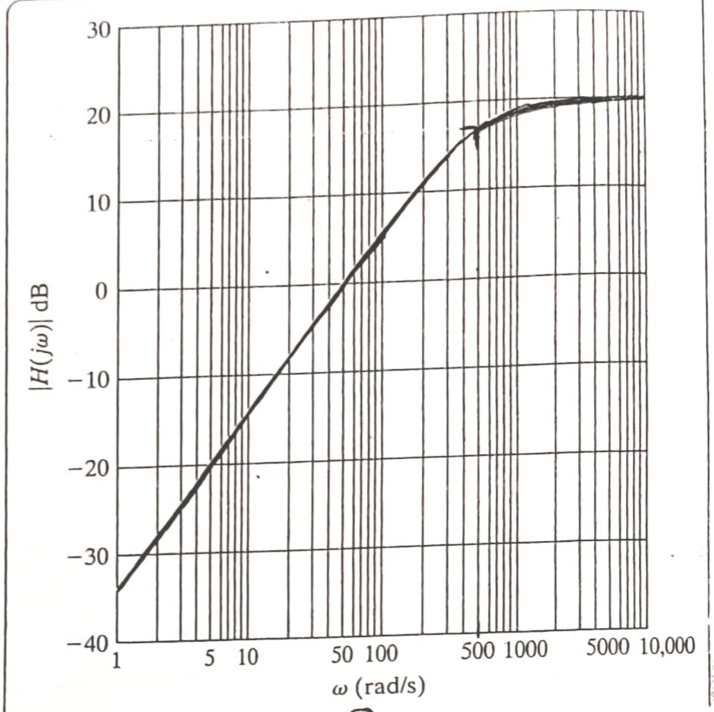
$R_1 = \frac{R_2}{5} = \frac{15915.5}{5} = 3.183\text{ k}\Omega$ $\qquad H(s) = -k \frac{w_c}{s+w_c}$

$R_1 = 3.183\text{ k}\Omega$, $R_2 = 15.9155\text{ k}\Omega$, $C = 0.01\,\mu F \Rightarrow H(s) = \frac{-31415.93}{s+6283.185}$



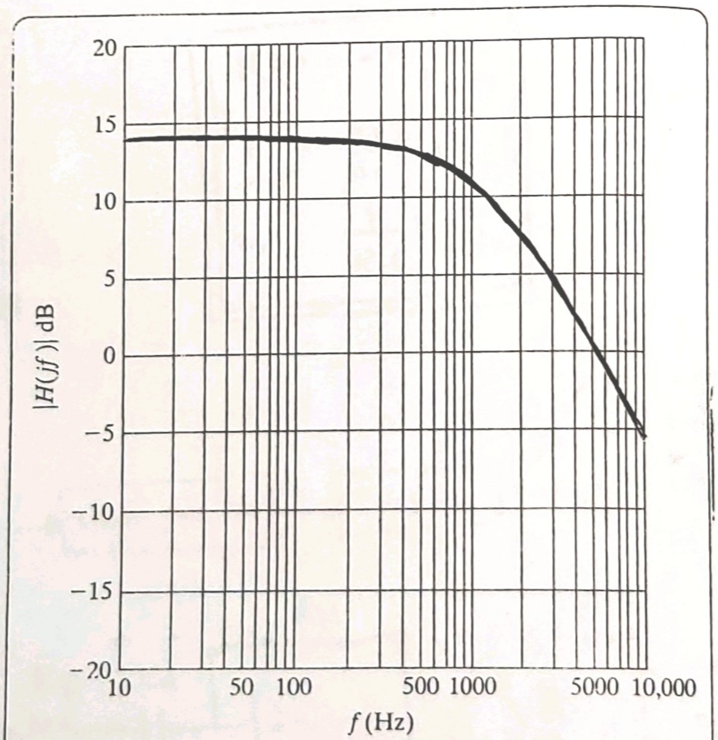
10
5
0
−5
−10
−15
−20
|H(jω)| dB
0.1
0.5
1.0
5.0
10
ω (rad/s)

1



30
20
10
0
−10
−20
−30
−40
|H(jω)| dB
1
5
10
50
100
500
1000
5000
10,000
ω (rad/s)

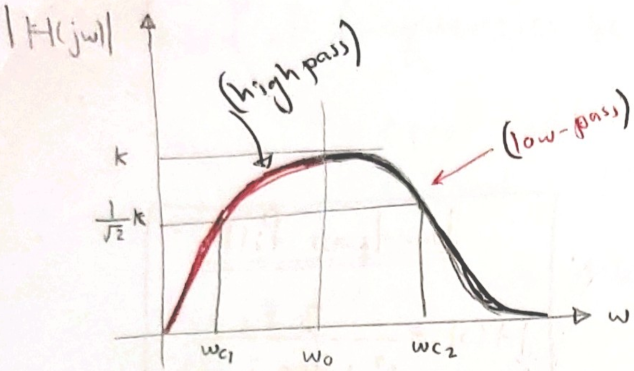
2



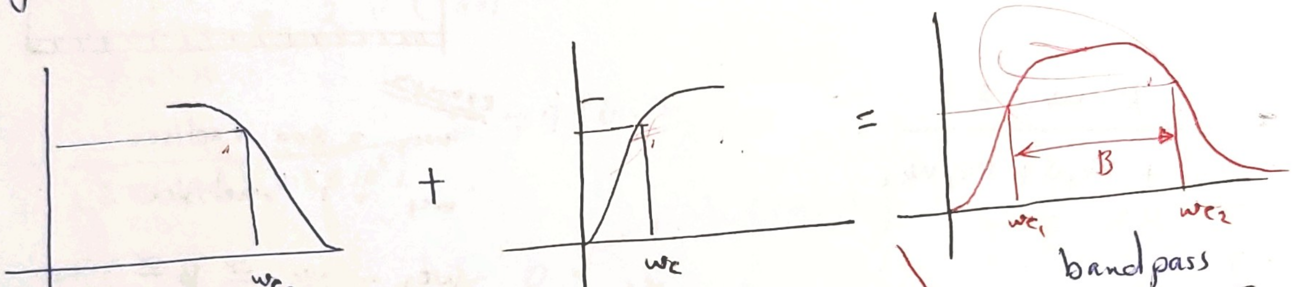
20
15
10
5
0
−5
−10
−15
−20
|H(jf)| dB
10
50
100
500
1000
5000
10,000
f (Hz)

3

3 Active Bandpass Filter :-



1. A unity-gain low-pass filter with cutoff freq ω_{c2}.
2. A unity gain high-pass filter with cutoff freq ω_{c1}.
3. A gain stage $\Rightarrow$ Passband gain $= k$



low-pass

$$\boxed{\begin{aligned} H(s) &= -k\,\frac{\omega_c}{s+\omega_c} \\ k &= \frac{R_2}{R_1} \\ \omega_c &= \frac{1}{R_2 C} = \frac{1}{R_L C_L} \end{aligned}}$$

high pass

$$\boxed{\begin{aligned} H(s) &= -k\,\frac{s}{s+\omega_c} \\ k &= \frac{R_2}{R_1} \\ \omega_c &= \frac{1}{R_1 C} = \frac{1}{R_H C_H} \end{aligned}}$$

bandpass

$\boxed{\omega_{c2} > \omega_{c1}}$ Important condition.

- broadband bandpass filter

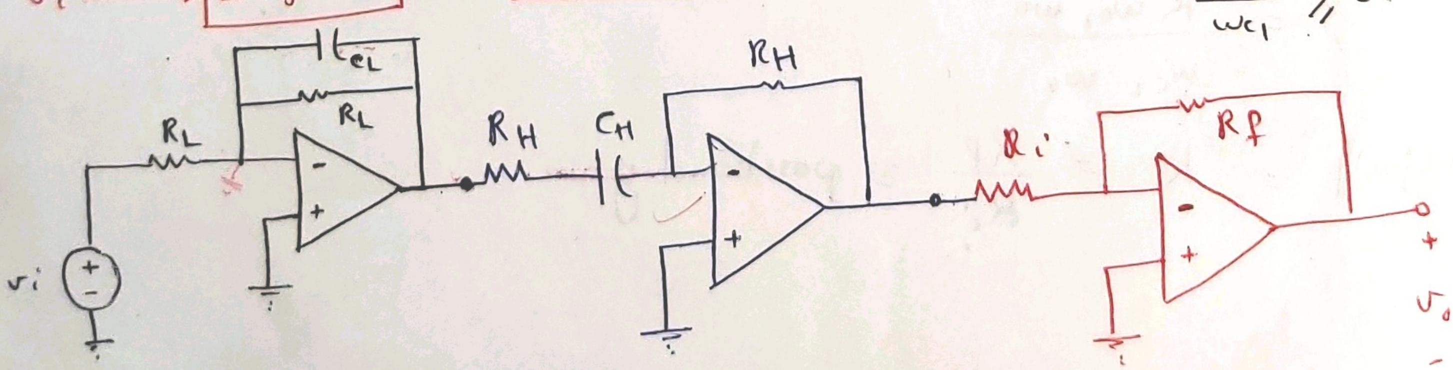
formal defn

- $\frac{\omega_{c2}}{\omega_{c1}} \geq 2$

The formal definition of a broadband filter requires the two cutoff frequencies to satisfy the equation

$$\frac{\omega_{c2}}{\omega_{c1}} \geq 2.$$

$v_i \rightarrow$ Low-pass filter $\rightarrow$ High-pass filter $\rightarrow$ Inv amp $\rightarrow v_o$



$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \left(\frac{-w_{c2}}{s+w_{c2}}\right)\left(\frac{-s}{s+w_{c1}}\right)\underbrace{\left(-\frac{R_f}{R_i}\right)}_{K}$$

$$= \frac{-K\, w_{c2}\, s}{(s+w_{c1})(s+w_{c2})}$$

$$= \frac{-K\, w_{c2}\, s}{s^2+(w_{c1}+w_{c2})s+w_{c1}w_{c2}}$$

$w_{c2} \gg w_{c1}$

~~broadband filter~~

$(w_{c1}+w_{c2}) \cong w_{c2}$

$$= \frac{-K\, w_{c2}\, s}{s^2+w_{c2}s+w_{c1}w_{c2}}.$$

bandpass filter

$$H(s) = \frac{\beta s}{s^2+\beta s+w_o^2}$$

$w_o = \sqrt{w_{c1} \cdot w_{c2}}$

$\beta = w_{c2} - w_{c1}$

$Q = \frac{w_o}{\beta}$

example

$w_{c2} = 500$ rad/sec

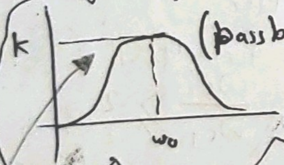
$w_{c1} = 1$ rad/sec

$w_{c2} - w_{c1} \cong \beta \cong w_{c2}$

$\boxed{1}$ $w_{c2} = \frac{1}{R_L C_L}$.

$\boxed{2}$ $w_{c1} = \frac{1}{R_H C_H}$.

$\boxed{3}$ K



(passband gain)

$$|H(jw)| = \left|\frac{-k\, w_{c2}(jw)}{(jw)^2+w_{c2}(jw)+w_{c1}w_{c2}}\right|_{w=w_o} = \left|\frac{-k\, w_{c2}\, w_o j}{\underbrace{-w_o^2}_{-w_{c1}w_{c2}}+w_{c2}w_o j+w_{c1}w_{c2}}\right|$$

$$= \frac{K\, \cancel{w_{c2}}\, \cancel{w_o}}{\cancel{w_{c2}}\, \cancel{w_o}}$$

$$|H(jw_o)| = K = \frac{R_f}{R_i} \equiv \text{passband gain.}$$

example

Design a bandpass filter to provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz. use 0.2 μF capacitors.

$w_{c1} = 2\pi(100)$ rad/sec

$w_{c2} = 2\pi(10,000)$ rad/sec

$w_{c2} = 100\, w_{c1} \Rightarrow w_{c2} \gg w_{c1}$

$K = 2$

[1] $w_{c2} = \dfrac{1}{R_L C_L} = 2\pi(10000) \Rightarrow R_L = \dfrac{1}{[2\pi(10,000)](0.2 \times 10^{-6})} \cong 80\,\Omega$

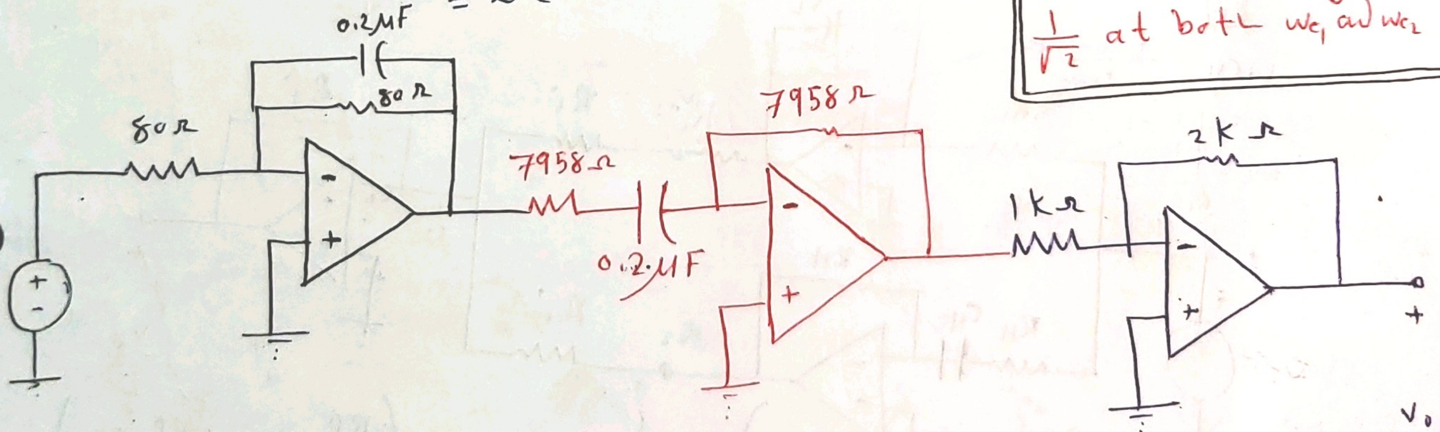
[2] $w_{c1} = \dfrac{1}{R_H C_H} = 2\pi(100)$

$R_H = \dfrac{1}{[2\pi(100)](0.2 \times 10^{-6})} \cong 7958\,\Omega$

$K = \dfrac{R_f}{R_i} \Rightarrow R_f = K R_i$ ← arbitrarily 1K

$= 2(1K) = 2K\Omega$

verify that the mag. of this ckT TF. is reduced by $\frac{1}{\sqrt{2}}$ at both w_{c1} and w_{c2}

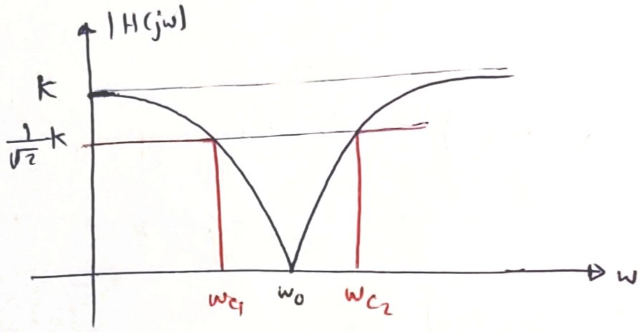


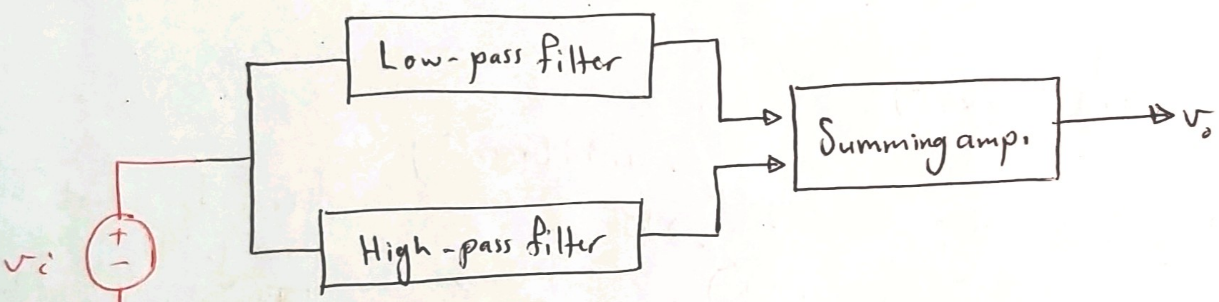
4 Active Bandreject Filter :-

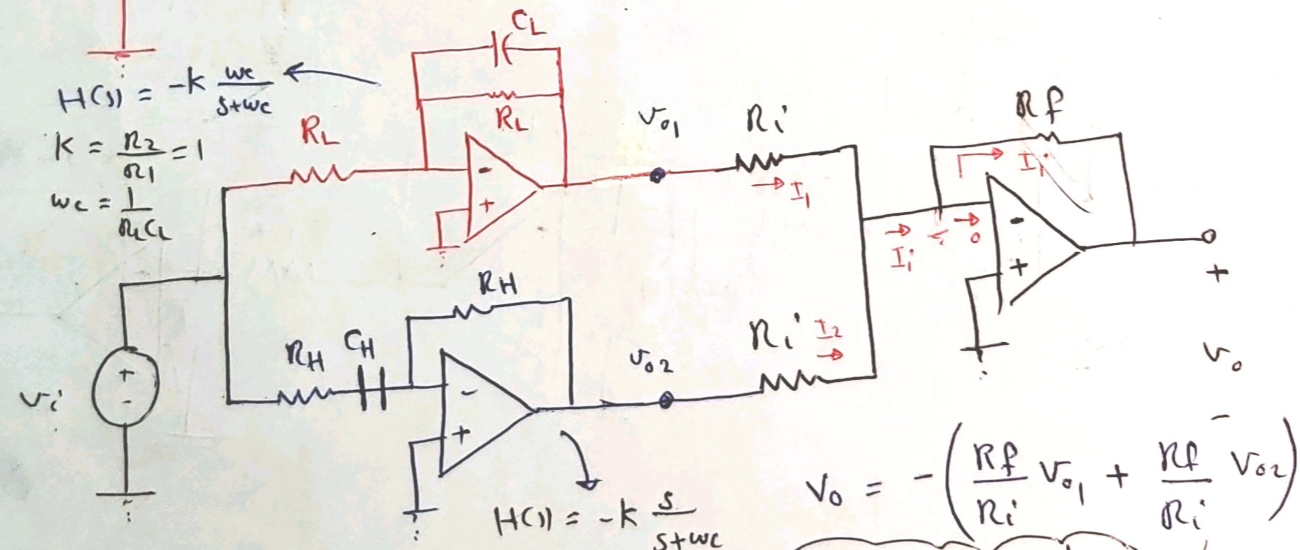
1. A unity gain low-pass filter with w_{c1}
2. A unity gain highpass filter with w_{c2}

$$w_{c_2} \gg w_{c_1}$$

3. gain stage







$H(s) = -k \frac{w_c}{s+w_c}$

$k = \frac{R_2}{R_1} = 1$

$w_c = \frac{1}{R_L C_L}$

$H(s) = -k \frac{s}{s+w_c}$

$k = \frac{R_2}{R_1} = 1$

$w_c = \frac{1}{R_H C_H}$

$$V_o = -\left(\frac{R_f}{R_i} V_{o1} + \frac{R_f}{R_i} V_{o2}\right)$$

Inverting adder

derive

$I_1 = \frac{V_{o1}}{R_i}$

$I_2 = \frac{V_{o2}}{R_i}$

$I_i = I_1 + I_2$

$V_o = -R_f I_i$

$$\boxed{V_o = -R_f\left(\frac{V_{o1}}{R_i} + \frac{V_{o2}}{R_i}\right)}$$

$$H(s) = -\left(\frac{R_f}{R_i}\right)\left[\frac{-w_{c1}}{s+w_{c1}} + \frac{-s}{s+w_{c2}}\right]$$

$$= \frac{R_f}{R_i}\left[\frac{w_{c1}(s+w_{c2}) + s(s+w_{c1})}{(s+w_{c1})(s+w_{c2})}\right]$$

$$= \frac{R_f}{R_i}\left[\frac{s^2 + 2w_{c1}s + w_{c1}w_{c2}}{(s+w_{c1})(s+w_{c2})}\right]$$

$s^2 + (w_{c1}+w_{c2})s + w_{c1}w_{c2}$

bandreject filter

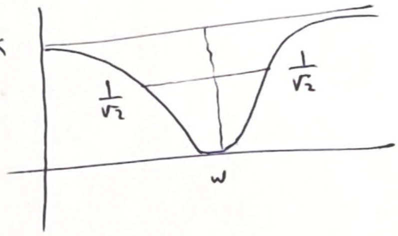
$$H(s) = \frac{s^2 + w_0^2}{s^2 + \beta s + w_0^2}$$

$w_0 = \sqrt{w_{c1}w_{c2}}$

$\boxed{1}$ $w_{c1} = \dfrac{1}{R_L C_L}$

$\boxed{2}$ $w_{c2} = \dfrac{1}{R_H C_H}$

$\dfrac{R_f}{R_i} = k$



as $s \to 0$

$s \to \infty$

$$|H(j\omega)|_{\omega=0} = k = \frac{R_f}{R_i}$$

$$|H(j\omega)|_{\omega=\infty} = k = \frac{R_f}{R_i}$$

$$|H(j\omega)|_{\omega=\omega_0} = \frac{R_f}{R_i}\left|\frac{(j\omega_0)^2 + 2w_{c1}\cancel{(j\omega_0)} + w_{c1}w_{c2}}{(j\omega_0)^2 + (w_{c1}+w_{c2})\cancel{(j\omega_0)} + w_{c1}w_{c2}}\right|$$

$$= \frac{R_f}{R_i}\,\frac{2w_{c1}}{w_{c1}+w_{c2}} \cong \frac{R_f}{R_i}\cdot\frac{2w_{c1}}{w_{c2}} \cong \text{very small}$$

Design :- 6 unknowns

3 eqs

$C \checkmark \Rightarrow R_H \quad w_{c1}$

$R_L \quad w_{c2}$

$k = \dfrac{R_f}{R_i}$ (assume 1)